Algebra 8.6 Simple Equations

1. Introduction

Let's talk about the truth:

$$2 = 2$$ This is a true statement

What else can we say about 2 that is true?

Example 1

$2 = 2$

$1 + 1 = 2$

$2 \times 1 = 2$

$4 \times \frac{1}{2} = 2$

$4 - 2 = 2$

$$4 = 4$$ You try – make a few statements about 4 that are true.

Let's say you've made a true statement:

$$2 = 2$$

What if something happens to one of your numbers?

Add something as simple as a 1... $$2 + 1 \neq 2$$ ...and your statement is no longer true.

One way to respond is to accept what's happened to your number, and change your statement to make it true again:

$$2 + 1 = 2 + 1$$

This is called **balancing**:

If you start with a true statement... $$a = b$$

...and then something happens to one side... $$a + 1 = b + 1$$ ...then, if you want the statement to stay true, you have to make the same thing happen to the other side.

Keeping a statement balanced and true is easy so long as you reflect any changes that happen to one side of the statement on the other side:

Example 2

Make sure each statement stays true, no matter what happens to the a

$x = 2$	$a = 3$
$x + 1 = 2 + 1$	$a + 2 =$
$x - 3 = 2 - 3$	$a - 4 =$
$3x = 3(2)$	$4a =$
$\frac{x}{2} = \frac{2}{2}$	$\frac{a}{4} =$
$xy = 2y$	$ab =$
$x^2 = 2^2$	$a^3 =$
$-x = -2$	$-a =$

If x gets multiplied by 3, then 2 has to be multiplied by 3 as well...

If x is turned into a negative number, then 2 has to become a negative as well...

$= 3+2,\ 3-4,\ 4(3),\ \frac{3}{4},\ 4b,\ 3^3,\ -3$

Here are some easy statements to keep balanced:

Problem 1

If $3 = 3$

then $3 + 2 =$

Problem 2

If $4 = 4$

then $4 - 9 =$

Problem 3

If $x = x$

then $x + 2 =$

Problem 4

If $x = x$

then $x - 6 =$

Problem 5

If $a = b$

then $3 - a =$

Problem 6

If $y = z$

then $4y^2 =$

But what if the statements are more complicated, for instance have more terms?

$$x + 6 = x + 6$$

What if something happened to this statement?

2. Balancing statements with more terms

Example 1

If $x+6\overset{+2}{=}x+6$

$x+6+2=x+6+\underline{\underline{2}}$

$x+8=x+8$

If 2 gets added to one side, you have to add it to the other to keep the statement true.

Once you've done that, you can simplify by grouping the like terms.

If $a+4\overset{+3}{=}a+4$

$a+4+3$

$a+7=a+7$

Example 2

If $x-2\overset{+5}{=}x-2$

$x-2+5=x-2+\underline{\underline{5}}$

$x+3=x+3$

If $a-3\overset{+6}{=}a-3$

$a-3+6$

$a+3=a+3$

Example 3

If $x+2\overset{-6}{=}x+2$

$x+2-6=x+2-6$

$x-4=x-4$

If $a+1\overset{-3}{=}a+1$

$a+1-3$

$a-2=a-2$

Example 4

If $\overset{x2}{x}+3=x+3$

$2(x)+3=2(x)+3$

$2x+3=2x+3$

If $\overset{x3}{a}+2=a+2$

$3(a)+2$

$3a+2=3a+2$

Example 5

If $\overset{x2}{x}+\overset{x2}{3}=x+3$

$2(x+3)=2(x+3)$

$2x+6=2x+6$

If $\overset{x3}{a}+\overset{x3}{2}=a+2$

$3(a+2)$

$3a+6=3a+6$

Example 6

If $\overset{\div 2}{x}+\overset{\div 2}{3}=x+3$

$\dfrac{x+3}{2}=\dfrac{x+3}{2}$

Since neither x or 3 are divisible by 2, we can't simplify this statement.

If $\overset{\div 3}{a}+\overset{\div 3}{2}=a+2$

$\dfrac{a+2}{3}$

$\dfrac{a+2}{3}=\dfrac{a+2}{3}$

3. Balancing equations

So far you've balanced statements where both sides are exactly the same:

$$x+3=x+3$$

But of greater interest are statements where both sides are not the same:

$$x+3=5$$

If you made a statement like this, and something happened to change one side, how do you balance the other side to keep the statement true?

Example 1

$$x+3\overset{+2}{=}5$$

$$x+3+2=5+2$$

If 2 gets added to one side, you have to add it to the other to keep the statement true

$$x+5=7$$

Once you've done that, you can simplify by grouping the like terms

$$a+4\overset{+3}{=}6$$

$$a+4+3$$

$a+7=9$

Example 2

$$x+2\overset{-6}{=}5$$

$$x+2-6=5-6$$

$$x-4=-1$$

$$a+4\overset{-7}{=}6$$

$$a+4-7$$

$a-3=-1$

Example 3

$$2y-2\overset{+4}{=}4$$

$$2y-2+4=4+4$$

$$2y+2=8$$

$$3a-4\overset{+6}{=}3$$

$$3a-4+6$$

$3a+2=9$

So far everything might seem easy. But consider this:

Example 4

Here we are multiplying the x by 2...

$$\overset{\times 2}{x}+3=5$$

$$2(x)+3\neq 2(5)$$

...but doubling part of one side of the equation is not the same as doubling the whole of the other side.

$$\overset{\times 3}{a}+2=7$$

$$3(a)+2$$

For instance: $2+3=5$
But $2(2)+3=7\neq 2(5)$

$3(a)+2\neq 3(7)$

3. Balancing equations, continued

If you multiply or divide anything in an equation, you can only keep both sides balanced and true if you **multiply and divide all terms on both sides the same way**.

Example 5

$\overset{\times 2}{x}+\overset{\times 2}{3}=5$

If you multiply one term by 2, then you have to multiply all terms by 2

$2(x+3)=2(5)$

$2x+6=10$

$\overset{\times 2}{a}+\overset{\times 2}{3}=7$

$2(a+3)$

$2a+6=14$

Example 6

$\overset{\div 2}{x}+\overset{\div 2}{3}=9$

+4

$\dfrac{x+3}{2}=\dfrac{9}{2}$

$\overset{\div 5}{a}+\overset{\div 5}{7}=12$

$\dfrac{a+7}{5}=\dfrac{12}{5}$

This is the hardest thing about balancing: making sure you do the same thing on both sides of an equation.

Use the same kind of care you would use when defusing a bomb.

Problem 1

$x+\overset{+4}{5}=4$

If you add 4 to one side then...

$x+9=8$

Problem 2

$a+\overset{+9}{3}=7$

$a+12=16$

Problem 3

$x+\overset{-4}{12}=18$

$x+8=14$

Problem 4

$y+\overset{-6}{7}=19$

$y+1=13$

Problem 5

$m+\overset{-7}{5}=12$

$m-2=5$

Problem 6

$m-\overset{-6}{3}=11$

$m-9=5$

3. Balancing equations, continued

Problem 7

$x + 4 \overset{+4}{=} -2$

$x+8=2$

Problem 8

$a + 7 \overset{+6}{=} -3$

$a+13=3$

Problem 9

$4 + a \overset{+7}{=} -5$

$11+a=2$

Problem 10

$6 - a \overset{+2}{=} -3$

$8-a=-1$

Problem 11

$14 - 2x \overset{-7}{=} 12$

$7-2x=5$

Problem 12 Remember: multiply **all** terms.

$x + 3 \overset{\times 2}{=} 7$

$2x+6=14$

Problem 13

$3 - x \overset{\times 4}{=} -4$

$12-4x=-16$

Problem 14

$2m - 4 \overset{\times 3}{=} 11$

$6m-12=33$

Problem 15 And divide **all** terms, too.

$x + 5 \overset{\div 4}{=} 7$

$\frac{x+5}{4}=\frac{7}{4}$

Problem 16

$y + 7 \overset{\div 6}{=} 19$

$\frac{y+7}{6}=\frac{19}{6}$

Problem 17 You'll be able to simplify this one.

$m + 5 \overset{\div 6}{=} 12$

$\frac{m+5}{6}=2$

Problem 18

$m - 3 \overset{\div 3}{=} 11$

$\frac{m}{3}-1=\frac{11}{3}$

Problem 19

÷ 2

$$2x+4=6$$

$x+2=3$

Problem 20

÷ 3

$$6-3a=4$$

$2-a=\frac{4}{3}$

Problem 21

÷ 6

$$6-3a=4$$

$1-\frac{a}{2}=\frac{2}{3}$

Problem 22

÷ 4

$$4m+14=16$$

$m+\frac{7}{2}=4$

Problem 23

x 2

$$\frac{x}{2}+3=6$$

$x+6=12$

Problem 24

x 3

$$\frac{y}{3}-4=6$$

$y-12=18$

Problem 25

x 3

$$\frac{2y}{3}-3=4$$

$2y-9=12$

Problem 26

x 3y

$$\frac{2x}{3y}-2=5$$

$2x-6y=15y$

Problem 27

x a

$$ab+2=12$$

$a^2b+2a=12a$

Problem 28

x 2m

$$2m-3=6$$

$4m^2-6m=12m$

Problem 29

^2

$$2+x=3$$

This means "raised to the power of 2"

$4+x^2=9$

Problem 30

^2

$$2-x=-2$$

$4+x^2=4$

Problem 31

$x + 4 \overset{-4}{=} 6$

$x=2$

Problem 32

$x + 9 \overset{-9}{=} 13$

$x=4$

Problem 33

$2x + 4 \overset{-4}{=} 6$

$2x=2$

Problem 34

$2x \overset{\div 2}{=} 2$

$x=1$

Problem 35

$3x \overset{\div 3}{=} 6$

$x=2$

Problem 36

$\frac{x}{2} \overset{\times 2}{=} 4$

$x=8$

Problem 37

$mn \overset{\div n}{=} 7$

$m=\frac{7}{n}$

Problem 38

$ab \overset{\div b}{=} 5b$

$a=5$

Problem 38

$x^2 \overset{\div x}{=} 9x$

$x=9$

Problem 39

$3m^3 + 3m \overset{\div 3m}{=} 15m$

$m^2+1=5$

Problem 40

$5x - 4 \overset{+4}{=} 11$

$5x=15$

Problem 41

$5x \overset{\div 5}{=} 15$

$x=3$

4. Solving simple equations

Why would you ever want to balance an equation?

How about when you need to find the value of a variable:

What number does this x represent?

$$x + 2 = 4$$

You can solve this easily by deducting 2 from each side:

$$x + 2 \overset{-2}{=} 4 \quad -2$$
$$x = 2$$

Solving an equation means finding the hidden value of a variable

The idea is to **get the variable by itself** on one side of the equation, and all the other information on the other side. This can mean using multiplication and division too:

Example 1

$$2x = 8$$
$$2x \overset{\div 2}{=} 8 \quad \div 2$$
$$x = 4$$

$$2a = 6$$

a=3

Sometimes you can solve an equation in one step:

Example 2

$$\frac{x}{3} = 4$$
$$\frac{x}{3} \overset{\times 3}{=} 4 \times 3$$
$$x = 12$$

$$\frac{a}{2} = 5$$

a=10

Often it will take more than one step:

Example 3

$$3x + 5 = 17$$
$$3x + 5 \overset{-5}{=} 17 \quad -5$$
$$3x \overset{\div 3}{=} 12 \quad \div 3$$
$$x = 4$$

$$2a + 5 = 17$$

a=6

Example 1

$$x - 5 = 8$$
$$x - 5 \overset{+5}{=} 8 + 5$$
$$x = 13$$

$$a - 5 = 7$$

$a=12$

Example 2

$$x - 6 = -4$$
$$x - 6 \overset{+6}{=} -4 + 6$$
$$x = 2$$

$$a - 6 = -2$$

$a=4$

Problem 1

$$x + 3 = 7$$

$x=4$

Problem 2

$$m + 12 = 10$$

$m=-2$

Problem 3

$$5 + x = 9$$

$x=4$

Problem 4

$$x - 6 = 13$$

$x=19$

Problem 5

$$x + 5 = 0$$

$x=-5$

Problem 6

$$5 - x = -2$$

$x=7$

Problem 7

$$x - 3 = 3$$

$x=6$

Problem 8

$$x - 3 = -3$$

$x=0$

Problem 9

$$x + 3 = 3$$

$x=0$

Problem 10

$$x - 12 = 15$$

$x=27$

6. Substituting numbers to check your answers

Seriously, when doing this kind of algebra there is no excuse for getting it wrong because you can always check your work by substituting numbers for variables:

Example 1

$$x - 3 = 3$$
$$x = 6$$
$$6 - 3 = 3$$
$$3 = 3$$

Plug the answer back into the original and check you get the same result.

$$a - 5 \overset{+5}{=} 7 \; {+5}$$
$$a = 12$$

That was an easy example. Here's what's coming:

Example 2

$$5x - 3 = -13$$
$$5x - \overset{+3}{3} = -13 \; {+3}$$
$$5\overset{\div 5}{x} = -10 \; {\div 5}$$
$$x = -2$$

After all that, you'll **really** want to check that you didn't muck up anything.

$$5(-2) - 3 = 13$$
$$-10 - 3 = -13$$
$$-13 = -13$$

That's good.

$$5x - 4 = -14$$

Or try this one on for size:

Example 3

$$\frac{x}{5} + 2 = 6$$
$$\frac{x}{5} + \overset{-2}{2} = 6 \; {-2}$$
$$\frac{x^{\times 5}}{5} = 4 \; {\times 5}$$
$$x = 20$$
$$\frac{20}{5} + 2 = 6$$

Plugging back in.

$$4 + 2 = 6$$
$$6 = 6$$

$$\frac{x}{4} + 3 = 6$$

Always check your work by plugging your answer back into the question.

7. Negative numbers

What do you do in this situation:

$$3 - x = 6$$
$$-x = 3$$

Do you leave the answer as −x?

Usually, and this is only a rule of thumb, when solving an equation you want to find the value for the positive version of the variable, not the negative.

$$3 - x = 6$$
$$-x = 3$$
$$x = -3$$

This is better

How do you get from one to the other? One way is to just remember that you can flip positive and negative numbers around like this:

Example 1

$$-y = 5$$
$$y = -5$$

$$-a = 12$$

a=−12

Example 2

$$-y = -5$$
$$y = 5$$

$$-a = -4$$

a=4

But if you want to know **why** you can do this, it's simple: you're just multiplying (or dividing) both sides of the statement by -1:

Example 3

$$3 - x = 6$$
$$-x = 3$$
$$-1(-x) = -1(3)$$
$$x = -3$$

If you understand this step, it will make your life with negative numbers much easier.

$$7 - a = 9$$

a=−2

Example 4

$$3 - x = -6$$
$$-x = -9$$
$$-1(-x) = -1(-9)$$
$$x = 9$$

$$7 - a = -9$$

a=16

8. Solving simple equations of the form 2x=4

Example 1

$3x = 12$

$3x \overset{\div 3}{=} 12 \div 3$

$x = 4$

$4a = 12$

$a=3$

Example 2

$-5m = 20$

$-5m \overset{\div(-5)}{=} 20 \div(-5)$

$m = -4$

$-5m = 30$

$m=-6$

Problem 1

$4x = 24$

$x=6$

Problem 2

$3x = -21$

$x=-7$

Problem 3

$7x = 35$

$x=5$

Problem 4

$-6x = -12$

$x=2$

Problem 5

$\dfrac{x}{3} = -8$

$x=-24$

Problem 6

$-x = -100$

$x=100$

Problem 7

$-8x = -24$

$x=3$

Problem 8

$\dfrac{m}{2} = -7$

$m=-14$

Problem 9

$\dfrac{x}{2} = 5$

$x=10$

Problem 10

$\dfrac{x}{6} = 2$

$x=12$

9. Solving simple equations of the form 2x+1=5

Example 1

$$5x - 3 = -13$$
$$5x - 3 \overset{+3}{=} -13 \;\; +3$$
$$5x \overset{\div 5}{=} -10 \;\; \div 5$$
$$x = -2$$

$$2a - 3 = -13$$

$a = -5$

Example 2

$$6 - 3x = 9$$
$$6 - 3x \overset{-6}{=} 9 \;\; -6$$
$$-3x \overset{\div(-3)}{=} 3 \;\; \div(-3)$$
$$x = -1$$

$$-5m = 30$$

$a = -6$

Problem 1

$$2x + 7 = 17$$

$x = 5$

Problem 2

$$4x + 4 = 16$$

$x = 3$

Problem 3

$$5 + 2y = 17$$

$y = 6$

Problem 4

$$6 - 3a = 18$$

$a = -4$

Problem 5

$$6 - 3x = 9$$

$x = -1$

Problem 6

$$6y - 7 = -19$$

$y = -2$

Problem 7

$$9 - 8x = 33$$

$x = -3$

Problem 8

$$7 - 2x = 1$$

$x = 3$

9. Solving simple equations of the form 2x+1=5, continued

Problem 9

$-6-4m=-10$

$m=1$

Problem 10

$4-3x=-2$

$x=2$

Problem 11

$-3+8x=13$

$x=2$

Problem 12

$-5-3x=7$

$x=-4$

Problem 13

$8-5a=-17$

$a=5$

Problem 14

$21-3x=12$

$x=3$

10. Solving simple equations of the form x⁄2+1=3

Example 1

$$\frac{x}{2} - 7 = -5$$

$$\frac{x}{2} - 7 \overset{+7}{=} -5 \; +7$$

Always take care with negative numbers!

$$\frac{x}{2} \overset{\times 2}{=} 2 \; \times 2$$

$$x = 4$$

$$\frac{a}{2} - 5 = -3$$

a=4

Example 2

$$\frac{x}{5} + 2 = 6$$

$$\frac{x}{5} + 2 \overset{-2}{=} 6 \; -2$$

$$\frac{x}{5} \overset{\times 5}{=} 4 \; \times 5$$

$$x = 20$$

$$\frac{y}{4} + 2 = 6$$

y=16

Problem 1

$$\frac{x}{4} + 4 = 8$$

x=16

Problem 2

$$\frac{a}{2} - 8 = -5$$

a=6

Problem 3

$$\frac{y}{3} - 8 = -5$$

y=9

Problem 4

$$\frac{m}{2} + 6 = 13$$

m=14

Problem 5

$$\frac{x}{3} + 4 = 3$$

x=−3

Problem 6

$$\frac{x}{5} - 4 = -7$$

y=−15

11. Solving simple equations of the form 2x+1=x+2

What if you have a variable on **both** sides of the equation?

Example 1

$$4x+5=2x+13$$
$$4x+5\overset{-5}{=}2x+13\,{-5}$$
$$4x\overset{-2x}{=}2x+8\,{-2x}$$
$$2x\overset{\div 2}{=}8\,{\div 2}$$
$$x=4$$

Removing the 5 from the left side...

...and removing 2x from the right side

$$4a+1=a+13$$

a=4

Remember, the point is to **get one variable by itself on one side of the equation**.

Example 2

$$10x-11=2x-3$$
$$10x-11\overset{+11}{=}2x-3\,{+11}$$
$$10x\overset{-2x}{=}2x+8\,{-2x}$$
$$8x\overset{\div 8}{=}8\,{\div 8}$$
$$x=1$$

$$10a-14=4a-2$$

a=2

Problem 1

$$5x+2=x+10$$

x=2

Problem 2

$$8x-5=3x+10$$

x=3

Problem 3

$$6x+10=3x-32$$

x=−14

Problem 4

$$5x-12=3x+6$$

x=9

Problem 5

$9x+7=5x-5$

$x=-3$

Problem 6

$10x+10=3x+3$

$x=-1$

Problem 7

$a-4=3a-8$

$a=2$

Problem 8

$y+6=6y-9$

$y=3$

Problem 9

$x-3=4x-9$

$x=2$

Problem 10

$x-8=5x+4$

$x=-3$

Problem 11

$2x-13=11x-4$

$x=-1$

Problem 12

$5x-2=7x-12$

$x=5$

Problem 13

$2x-40=5x-10$

$x=-10$

Problem 14

$6x+13=4x+13$

$x=0$

12. Solving equations of the form 2(x+1)=4

Example 1

$$\begin{aligned} 2(x+3) &= 10 \\ 2x+6 &= 10 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

$$2(y+4)=12$$

$y=2$

Example 2

$$\begin{aligned} 3(2x-4) &= 18 \\ 6x-12 &= 18 \\ 6x &= 30 \\ x &= 5 \end{aligned}$$

$$4(x-5)=28$$

$x=12$

Problem 1

$$2(3x-4)=10$$

$x=3$

Problem 2

$$2(x+6)=6$$

$x=-3$

Problem 3

$$5(2x+9)=15$$

$x=-3$

Problem 4

$$2(5x-6)=-22$$

$x=-1$

Problem 5

$$3(x+4)=6$$

$x=-2$

Problem 6

$$3(2x-1)=9$$

$x=2$

12. Solving equations of the form 2(x+2)=3(x+1)

Example 1

$$4(x+3)+3(x-2)=34$$

$$4x+12+3x-6=34$$

Group like terms...

$$7x+6=34$$

$$7x=28$$

$$x=4$$

$$3(2x-3)+2(x-4)=-25$$

$x=-1$

Example 2

$$2(x-7)=6(x+1)$$

$$2x-14=6x+6$$

$$2x=6x+20$$

$$-4x=20$$

$$x=-5$$

$$3(x+4)=2(4x+1)$$

$x=2$

Problem 1

$$2(3x-4)+3(x+4)=-41$$

$x=-5$

Problem 2

$$5(2x+1)-3(x-3)=35$$

$x=3$

Problem 3

$$3(3x-4)-2(2x-3)=-11$$

$x=-1$

Problem 4

$$3(3x+4)=2(x+13)$$

$x=2$

Problem 5

$$7(x+2)=3x+10$$

$x=-1$

Problem 6

$$4(x+3)=-2(x+6)$$

$x=-4$

13. Mixed practice

Problem 1

$$x + 9 = 2$$

$x = -7$

Problem 2

$$x + 7 = 12$$

$x = 5$

Problem 3

$$x + 5 = -3$$

$x = -8$

Problem 4

$$x - 5 = -8$$

$x = -3$

Problem 5

$$\frac{-x}{2} = 6$$

$x = -12$

Problem 6

$$\frac{-x}{3} = -4$$

$x = 12$

Problem 7

$$8x = 48$$

$x = 6$

Problem 8

$$-x = -1$$

$x = 1$

Problem 9

$$\frac{x}{2} - 3 = -7$$

$x = -8$

Problem 10

$$\frac{a}{3} + 9 = 8$$

$a = -3$

Problem 11

$$\frac{m}{2} - 5 = -8$$

$m = -6$

Problem 12

$$\frac{x}{5} - 9 = -5$$

$x = 20$

13. Mixed practice, continued

Problem 13

$2(2x+3)=20$

$x=\frac{7}{2}$

Problem 14

$11(x-4)=33$

$x=7$

Problem 15

$3(x+3)=-27$

$x=-12$

Problem 16

$3(y-6)=4(y+4)$

$y=-34$

Problem 17

$5(2x+3)-4(x+2)=19$

$x=2$

Problem 18

$2(4x-3)=3(x-7)$

$x=-3$